Table 2 Effect of specular reflection on heat flux[a]

		Dimensionless heat flux $q(\tau_1)/\sigma(T_1^4 - T_2^4)$					
		Isotropic scattering		Forward scattering		Backward scattering	
N_c	ω	Diffuse[b]	Specular[c]	Diffuse	Specular	Diffuse	Specular
0.001	0.1	0.0729	0.0709	0.0730	0.0710	0.0728	0.0708
	0.4	0.0662	0.0647	0.0666	0.0650	0.0659	0.0644
	0.9	0.0423	0.0420	0.0425	0.0422	0.0421	0.0418
	1	0.0255	0.0255	0.0255	0.0255	0.0255	0.0255
0.01	0.1	0.1967	0.1902	0.1972	0.1906	0.1962	0.1898
	0.4	0.1717	0.1676	0.1729	0.1688	0.1704	0.1664
	0.9	0.0985	0.0980	0.0989	0.0985	0.0980	0.0976
	1	0.0722	0.0722	0.0722	0.0722	0.0722	0.0722
0.1	0.1	0.7189	0.7071	0.7199	0.7080	0.7179	0.7062
	0.4	0.6739	0.6669	0.6765	0.6691	0.6715	0.6646
	0.9	0.5674	0.5669	0.5680	0.5674	0.5669	0.5664
	1	0.5397	0.5397	0.5397	0.5397	0.5397	0.5397

[a] $\varepsilon_1 = \varepsilon_2 = 0.03$, $\tau_2 - \tau_1 = 1.0$, $r_1/r_2 = 0.5$, $\theta_2 = 0.1$, $\nu = 0.0$. [b] $\rho^s = 0$, $\rho^d = \rho$. [c] $\rho^d = 0$, $\rho^s = \rho$.

displayed the lowest heat flux results. This observation is useful in selecting filler materials for multilayer insulations for cryogenic and space applications. It is interesting to note from Table 2 that fully specularly reflecting ($\rho^d = 0$, $\rho^s = \rho$) boundaries always transfer less heat than fully diffusely reflecting ($\rho^s = 0$, $\rho^d = \rho$) boundaries. This difference diminishes as ω and N_c increase. However, diffuse and specular results do not show any appreciable deviation. The effect of variable thermal conductivity is also shown in Figs. 1–3. For the case of thermal conductivity increasing with temperature, the heat transfer rate is less as compared to the opposite case. However, at lower values of N_c, the effect of variable conductivity diminishes slightly as ω increases.

Conclusions

1) The effect of anisotropy is significant, particularly for the case of low conduction–radiation numbers and high values of scattering albedo. At all values of conduction–radiation numbers, strongly backward-scattering media transfers the least heat and also affects a reducing heat transfer with increasing scattering albedo.

2) Specular or diffuse boundaries make little difference in heat transfer, although the former transfer less heat.

3) The effect of variable thermal conductivity is significant.

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References

[1]Viskanta, R., "Radiation Transfer: Interaction with Conduction and Convection and Approximate Methods in Radiation," *Proceedings of the 7th International Heat Transfer Conference* (Munich, Germany), Vol. 1, 1982, pp. 103–121.

[2]Howell, J. R., "Thermal Radiation in Participating Media: The Past, the Present, and Some Possible Futures," *Journal of Heat Transfer*, Vol. 110, 1988, pp. 1220–1229.

[3]Fernandes, R., and Francis, J., "Combined Conductive and Radiative Heat Transfer in and Absorbing, Emitting, and Scattering Cylindrical Medium," *Journal of Heat Transfer*, Vol. 104, 1982, pp. 594–601.

[4]Pandey, D. K., "Combined Conduction and Radiation Heat Transfer in Concentric Cylindrical Media," *Journal of Thermophysics and Heat Transfer*, Vol. 3, No. 1, 1989, pp. 75–82.

[5]Harris, J. A., "Solution of the Conduction/Radiation Problem with Linear-Anisotropic Scattering in an Annular Medium by the Spherical Harmonics Method," *Journal of Heat Transfer*, Vol 111, 1989, pp. 194–197.

[6]Modest, M. F., *Radiative Heat Transfer*, McGraw–Hill, New York, 1993.

[7]Chu, H. S., and Tseng, C. J., "Conduction-Radiation Interaction in Absorbing, Emitting and Anisotropically Scattering Media with Variable Thermal Conductivity," *Journal of Thermophysics and Heat Transfer*, Vol. 6, No. 3, 1992, pp. 537–540.

[8]Hall, G., and Watt, J. M. (ed.), *Modern Numerical Methods for Ordinary Differential Equations*, Clarendon, Oxford, England, UK, 1976.

[9]Brewster, M. Q., *Thermal Radiative Transfer and Properties*, Wiley, New York, 1992.

Air High-Enthalpy Stagnation Point Heat Flux Calculation

F. De Filippis* and M. Serpico†
Centro Italiano Ricerche Aerospaziali,
Via Maiorise 81043 Capua, Italy

Introduction

A SPACE vehicle re-entering into the atmosphere is subject to very high thermal loads, because of the conduction and diffusion of energy through the boundary layer and, naturally, the stagnation region is the more stressed zone. Since 1950, hypersonic researchers have made great efforts to find analytical formulas to evaluate the heat-flux peak at the stagnation point. The theory developed by Fay and Riddell[1] in 1958 represents the most important milestone in this field. In their work Fay and Riddell solve the self-similar boundary-layer equations, taking into account the main chemical–physical effects in the air at high enthalpy (in particular dissociation and diffusion) for equilibrium or frozen boundary layer. The limited computer resources in 1958 imposed a great number of approximations. The viscosity and the thermal conductivity were modeled using the Sutherland law extrapolation at high temperature; the diffusivity was modeled assuming a constant value of the Prandtl number and considering the air mixture composed of only two species: molecular and atomic types. The final formulas proposed by Fay and Riddell, that are still in use today, permit the evaluation of the stagnation point heat flux starting from the flow properties at the external boundary-

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*Research Scientist, Plasma Wind Tunnel Section. Member AIAA.
†Ph.D., Research Engineer, Aerothermodynamics Section.

layer edge (known, for example, by means of a preliminary Eulerian calculations).

However, a simplified equation of Fay and Riddell formulas (obtained assuming approximate values of Lewis and Prandtl numbers) is currently used in aerospace applications.

In this work the re-examination of the stagnation point theory has been carried out starting from some available computational fluid dynamics (CFD) simulations of the flowfield around a model in the test chamber of Scirocco, the high-enthalpy Italian wind tunnel (70 MW). The code adopted is a hypersonic flow simulator that solves the full Navier–Stokes equations for air in thermal–chemical nonequilibrium, with the diffusion coefficients modeled by using the collision integral theory. All of the numerical heat flux values have been compared with the Fay and Riddell results and used to obtain a new correlation formula for an engineering prediction of the heat flux at the stagnation point.

Stagnation point theory

Fay and Riddell Formulas

The theory developed by Fay and Riddell[1] permits the calculation of the heat-flux at the stagnation point of a blunt body by resolution of the axisymmetric Navier–Stokes equations restricted to the boundary layer. Simplified transport and chemical models were used in their computations, assuming a fixed chemical state of the gas (frozen or equilibrium). The final formulas, reported hereafter, were obtained performing a parametric analysis of the heat flux on a re-entering body over a range of altitudes from 7.5 to 36 km and velocities from 1.8 to 7.0 km/s

Chemical equilibrium flow:

$$q_w = A[1 + (Le^{0.52} - 1)(h_D/h_{0_e})] \tag{1}$$

Frozen flow—fully catalytic wall:

$$q_w = A[1 + (Le^{0.63} - 1)(h_D/h_{0_e})] \tag{2}$$

Frozen flow—noncatalytic wall:

$$q_w = A[1 - (h_D/h_{0_e})] \tag{3}$$

with

$$A = 0.76Pr^{-0.6}(\rho_e\mu_e)^{0.4}(\rho_w\mu_w)^{0.1}\sqrt{\left(\frac{du_e}{dx}\right)_s}(h_{0_e} - h_w)$$

Pr is the Prandtl number, ρ_w is the density, μ_w is the viscosity coefficient, h_w is the enthalpy at the wall, h_{0_e} is the total enthalpy at the edge of the boundary layer, Le is the Lewis number, and $h_D = \Sigma_i\, c_{i_e}(\Delta h_f)_i^0$, with c_{i_e} being the concentrations at the edge of the boundary layer and $(\Delta h_f)_i^0$ being the heats of formation. The computations of Fay and Riddell were made with different Lewis numbers ranging from 1.0 to 2.0.

Approximate Formula

To evaluate the heat flux at the stagnation point of a blunt body, a simplified formula, derived from the Fay and Riddell equations,[1] is frequently used. In particular, this formula is used by some experimentalists[2,3] without taking into account the limitations caused by the maximum total enthalpy (<23 MJ/kg), by the constant Lewis and Prandtl number and by the simplified chemical-transport models. The formula, in the fully catalytic wall hypothesis, reads

$$q_w = K\sqrt{(p_e/R)}(h_{0_e} - h_w) \tag{4}$$

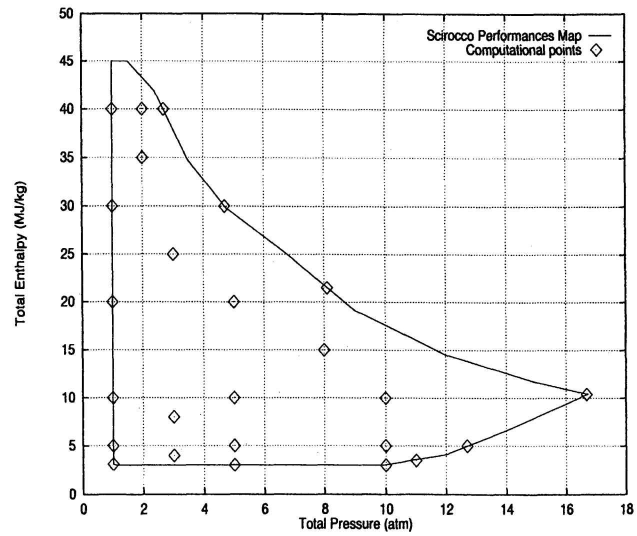

Fig. 1 Scirocco envelope map and computational points.

where K has a value of 112.4 (kg/s · cm$^{1.5}$ · atm$^{0.5}$), and with the pressure p_e in atm the radius R in cm, q_w in W/cm^2 and enthalpies h_{0_e} and h_w in MJ/kg.

CFD Results

The analysis of the stagnation point heat flux on a blunt body has been performed by Navier–Stokes calculations on a spherical model at the Scirocco Plasma Wind Tunnel (PWT) conditions (see Fig. 1). The Scirocco PWT is a 70-MW arc-heated hypersonic facility, currently in the construction phase at CIRA in Capua (Italy). The freestream properties in the test section of the Scirocco PWT have been obtained by means of a preliminary nonequilibrium, full Navier–Stokes computation of the nozzle flow expansion. For each nozzle reservoir condition, four different nozzle exit diameters were considered with sphere radius models of 0.3 and 0.24 m, in accordance with the Scirocco requirements.

Code Description

The research code used for the numerical simulation solves the full Navier–Stokes equations on structured grids for internal and external flows, in two-dimensional as well as in axisymmetric cases, with chemical and vibrational nonequilibrium. A finite volume technique has been used, with a flux difference splitting Riemann solver formulation,[4] with a second-order reconstruction of fluxes at cell interfaces. A detailed description of this solver is given in Refs. 5–7, where the code has been extensively tested and validated by computing different flowfields as shock-wave/boundary-layer interaction, base flow, and stagnation region.

The Park–Rakich chemical model[8] was used in the present computations, taking into account five chemical species (O, N, NO, O_2, and N_2); the transport model is the Yun–Mason one, where the transport coefficients are obtained by the theory developed by Chapman and Cowling.[9]

Computational Results

A preliminary analysis to determine the minimum grid size and stretching parameters for the heat flux grid independency has been performed. All of the computations have been obtained with the laminar flow hypothesis and fully catalytic wall. The numerically calculated heat flux variation along the sphere surface presented the classical stagnation point singularity, removed by a data extrapolation. This operation is correct because, for all analyzed cases, the singularity is confined in a small region near the stagnation region, where the heat flux variation is lower than 5% [$(q/q_0) = \cos^{3/2}\theta$].

The total enthalpy difference ($h_{0_e} - h_w$) varies from 2 to 39 MJ/kg, for which the effects of the chemical–nonequilibrium

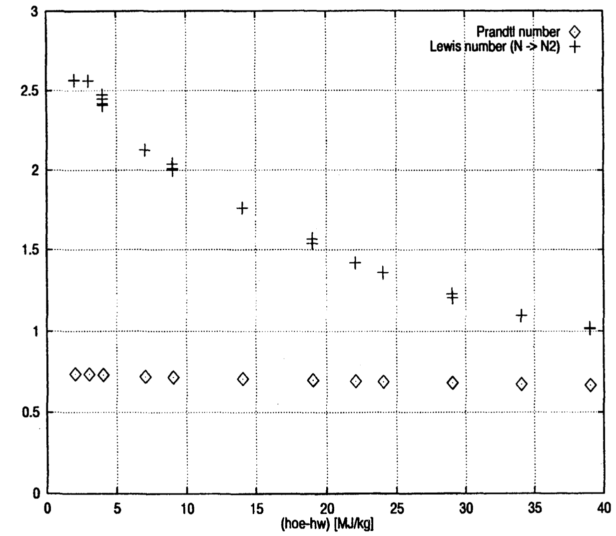

Fig. 2 Prandtl and Lewis number vs total enthalpy difference.

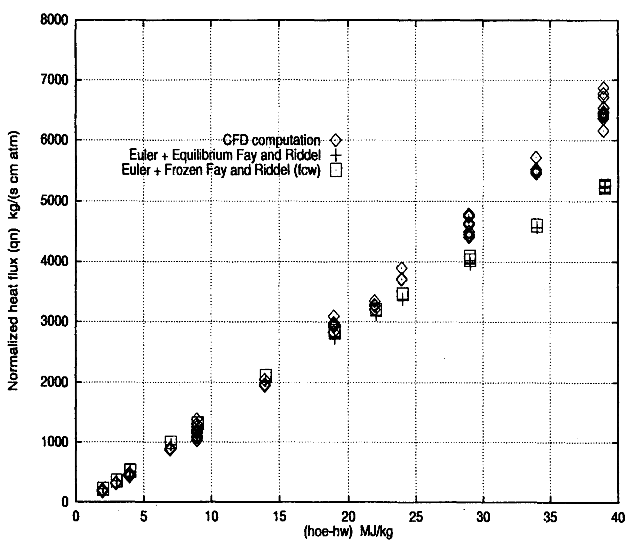

Fig. 3 Normalized heat flux at stagnation point: comparison between Fay and Riddell and CFD results.

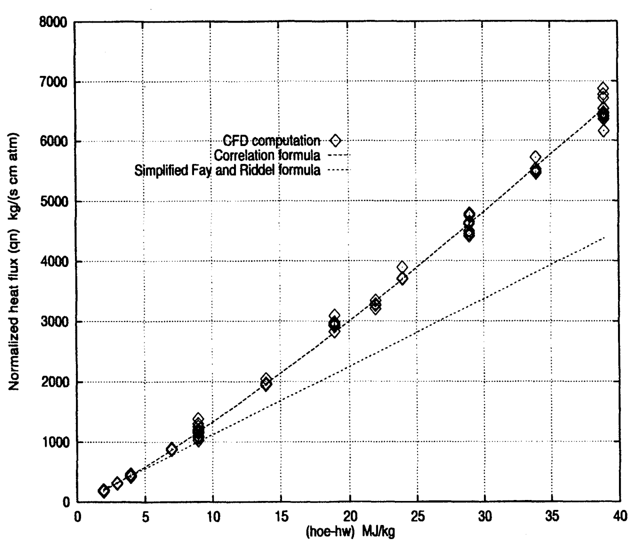

Fig. 4 Normalized heat flux at stagnation point: comparison between simplified Fay and Riddell and CFD correlation formulas.

and the transport properties variation with temperature are very important. These effects are clearly shown in Fig. 2, where the variations of the Lewis and Prandtl numbers are reported as a function of the enthalpy difference.

All of the numerical heat fluxes have been normalized with the factor $\sqrt{p_e/R}$ [$q_n = q_w\sqrt{(R/p_e)}$] and are shown in Figs. 3 and 4 as a function of the difference $(h_{0_e} - h_w)$. When observing Figs. 3 and 4 an almost linear variation of the Fay and Riddell results can be observed. It is relevant that at high enthalpies the slope of the fitting curve is equal to 170.6, which is very different from the value of 112.4 used in the approximate Fay and Riddell formula.

In Fig. 3 the CFD nonequilibrium results are compared with the equivalent one obtained from formulas (1) and (2). Looking at data reported in this figure, good agreement between CFD and Fay and Riddell[1] prevails up to an enthalpy difference of about 23 MJ/kg. This number represents the upper enthalpy limit of the Fay and Riddell analysis and confirms that in any case the extrapolation to higher-enthalpy values of the Fay and Riddell formulas is not accurate.

Starting from these data, a more accurate data fit that contains the normalized heat flux behavior at low- and high-enthalpy differences has been obtained. This relationship is

$$q_w = 90\cdot\sqrt{(p_e/R)}\cdot(h_{0_e} - h_w)^{1.17} \tag{5}$$

and it is shown in Fig. 4. In this relation the heat flux is obtained in W/cm^2, even using p_e in atm, R in cm, and the enthalpies (h_{0_e} and h_w) in MJ/kg. This correlation formula can be used in all engineering applications to predict the fully catalytic heat flux at the stagnation point of a blunt body.

Conclusions

In this work a new correlation formula to calculate the fully catalytic stagnation point heat flux has been derived from CFD computations. The comparison between the CFD and the Fay and Riddell theory shows good agreement up to a total enthalpy level of about 23 MJ/kg, which is the upper limitation postulated in the Fay and Riddell work. The present work demonstrates that for total enthalpies higher than 23 MJ/kg, extrapolation of the Fay and Riddell formulas is not accurate, whereas the new correlation formula is accurate for total enthalpies up to 39 MJ/kg.

References

[1]Fay, J. A., and Riddell, F. R., "Theory of Stagnation Point Heat Transfer in Dissociated Air, *Journal of the Aeronautical Sciences*, Vol. 25, No. 2, 1992, pp. 73–85.

[2]Winovich, W., and Carlson, W. C. A., "The 60-MW Shuttle Interaction Heating Facility," *Proceedings of the 25th International Instrumentation Symposium*, Instrument Society of America, ISA, Anaheim, CA, 1979.

[3]Tran, H. K., and Rasky, D. J., "Thermal Response and Ablation Characteristics of Lightweight Ceramic Ablators," *Journal of Spacecraft and Rockets*, Vol. 31, No. 6, 1994, pp. 993–998.

[4]Pandolfi, M., "On the Flux Difference Splitting Formulation," *Notes on Numerical Fluid Mechanics*, Vol. 24, Vieweg, Brunswick, Germany, 1989, pp. 466–481.

[5]De Filippis, F., Serpico, M., Marini, M., Ravachol, M., and Tribot, J. P., "Comparison Between Numerical and Experimental Results on Different HERMES Elevon Shapes," *Journal of Spacecracft and Rockets*, Vol. 34, No. 3, 1997, pp. 272–278.

[6]Serpico, M., Schettimo, A., Ciucci, A., Falconi, D., and Fabrizi, M., "VEGA Launcher Base Flow Prediction at Different Supersonic Mach Number," AIAA Paper 97-2302, June 1997.

[7]Serpico, M., Monti, R., and Savino, R., "Heat Flux on Partially Catalytic Surfaces in Hypersonic Flows," *Journal of Spacecracft and Rockets*, Vol. 35, No. 1, 1998, pp. 9–15.

[8]Rakich, J. V., Bailey, H. E., and Park, C., "Computation of Nonequilibrium, Supersonic Three-Dimensional Inviscid Flowfield over Blunt-Nosed Bodies," *AIAA Journal*, Vol. 21, No. 6, 1983, pp. 834–841.

[9]Hirschfelder, J. O., Curtiss, C. F., and Bird, R. B., *Molecular Theory of Gases and Liquids*, 2nd ed., Wiley, New York, 1963, Chap. 7.